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GEOMETRY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

68. Proposed by LEONARD E. DICKSON, M. A., Ph. D., Formerly Fellow of Mathematics, University of Chicago, Chicago, Illinois.

Suppose a circle of unit radius divided at the points A, A_1, A_2, A_3, ... into n equal parts. [This division cannot in general be affected by geometry.] Through A draw the diameter OA and join O with $A_1, A_2, A_3, \dots A_{\frac{n-1}{2}}$, where n is supposed to be odd.

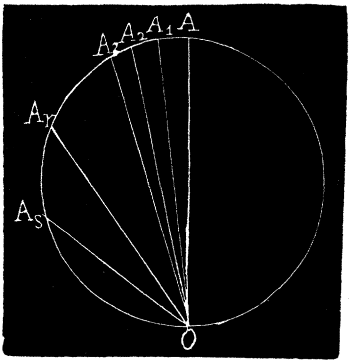
Prove that $OA_1-OA_2+OA_3-OA_4+\dots\dots\pm OA_{\frac{n-1}{2}}$, every other chord being affected with the minus sign.

Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas, and C. W. M. BLACK, A. M., Professor of Mathematics in Wesleyan Academy, Wilbraham, Massachusetts.

Let OA_1, OA_2, OA_3, etc. $=a_1$, a_2, a_3, etc.

Now $OA=2$, $\angle AOA_1=\angle A_1OA_2=\text{etc.}=\pi/n$.

$\therefore\ OA_r=a_r=2\cos(r\pi/n)$.



(1) When $\frac{n-1}{2}$ is even,

$$\therefore\ a_1+a_3+a_5+\dots\dots+a_{\frac{n-3}{2}}$$

$$=2\left(\cos\frac{\pi}{n}+\cos\frac{3\pi}{n}+\cos\frac{5\pi}{n}+\dots\dots\right.$$

$$\left.+\cos\frac{(n-3)\pi}{2n}\right)=\sin\frac{(n-1)\pi}{2n}\Big/\sin\frac{\pi}{n}$$

$$=\sin\left(\frac{\pi}{2}-\frac{\pi}{2n}\right)\Big/\sin\frac{\pi}{n}=\tfrac{1}{2}\cos\frac{\pi}{2n}\quad\dots\dots\ (1).$$

$$a_2+a_4+a_6+\dots\dots+a_{\frac{n-1}{2}}=2\left(\cos\frac{2\pi}{n}+\cos\frac{4\pi}{n}+\cos\frac{6\pi}{n}+\dots\dots+\cos\frac{(n-1)\pi}{2\tau}\right)$$

$$=\frac{2\cos[(n+3)\pi/4n]\sin[(n-1)\pi/4n]}{\sin(\pi/n)}=\frac{2\cos[\frac{1}{4}\pi+(3\pi/4n)]\sin[\frac{1}{4}\pi-(\pi/4n)]}{\sin(\pi/n)}$$

$$=\frac{[\cos(3\pi/4n)-\sin(3\pi/4n)][\cos(\pi/4n)-\sin(\pi/4n)]}{\sin(\pi/n)}=\frac{\cos(\pi/2n)-\sin(\pi/n)}{\sin(\pi/n)}$$

$$=\tfrac{1}{2}\operatorname{cosec}\frac{\pi}{2n}-1\dots\dots\ \dots\dots(2).$$

$$\therefore\ a_1-a_2+a_3-a_4+\ldots\ldots+a_{\frac{n-3}{2}}-a_{\frac{n-1}{2}}=1.$$

(2) When $\frac{n-1}{2}$ is odd.

$$\therefore\ a_1+a_3+a_5+\ldots\ldots+a_{\frac{n-1}{2}}=2(\cos\frac{\pi}{n}+\cos\frac{3\pi}{n}+\cos\frac{5\pi}{n}+\ldots\ldots$$

$$+\cos\frac{(n-1)\pi}{2n})=\sin\frac{(n+1)\pi}{2n}/\sin\frac{\pi}{n}$$

$$=\sin\left(\frac{\pi}{2}+\frac{\pi}{2n}\right)/\sin\frac{\pi}{n}=\tfrac{1}{2}\text{cosec}\frac{\pi}{2n}\ldots\ldots\ldots\ldots(3).$$

$$a_2+a_4+a_6+\ldots\ldots+a_{\frac{n-3}{2}}=2(\cos\frac{2\pi}{n}+\cos\frac{4\pi}{n}+\cos\frac{6\pi}{n}+\ldots.+\cos\frac{(n-3)\pi}{2n})$$

$$=\frac{2\cos[(n+1)\pi/4n]\sin[(n-3)\pi/4n]}{\sin(\pi/n)}=\frac{2\cos[\frac{1}{4}\pi+(\pi/4n)]\sin[\frac{1}{4}\pi-(3\pi/4n)]}{\sin(\pi/n)}$$

$$+\frac{[\cos(n/4n)-\sin(\pi/4n)][\cos(3\pi/4n)-\sin(3\pi/4n)]}{\sin(\pi/n)}=\frac{\cos(\pi/2n)-\sin(\pi/n)}{\sin(\pi/n)}$$

$$=\tfrac{1}{2}\text{cosec}(\pi/2n)-1.\ \ldots\ldots\ldots.(4).$$

$$\therefore\ a_1-a_2+a_3-a_4+\ldots\ldots-a_{\frac{n-3}{2}}+a_{\frac{n-1}{2}}=1.$$

$$\therefore\ OA_1-OA_2+OA_3-OA_4+\ \ldots..\pm OA_{\frac{n-1}{2}}=1.$$

70. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy in Ohio University, Athens, Ohio.

Prove that the locus of the center of the circle which passes through the vertex of a parabola and through its intersections with a normal chord is the parabola $2y^2=ax-a^2$, the equation to the given parabola being $y^2=4ax$.

Solution by the PROPOSER.

The circles being $(x-m)^2+(y-n)^2=r^2$........................(1),

and passing through the vertex of $y^2=4ax$.............. ....(2),

becomes $x^2-2mx+y^2-2ny=0$..(3).

Now the extremities of the normal chord being $(at_1{}^2,\ 2at_1)$, $(at_2{}^2,\ 2at_2)$, normal at the former point, we have

$$a^2t_1{}^4-2mt_1{}^2+4a^2t_1{}^2-4nat_1=0\ldots\ldots\ldots\ldots\ldots\ldots(4),$$

and $a^2t_2{}^4-2mt_2{}^2+4a^2t_2{}^2-4nat_2=0$..................(5).